

Baryon spectra with instanton induced forces

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(Dated: February 7, 2008)

Abstract

Except the vibrational excitations of K and K^* mesons, the main features of spectra of mesons composed of quarks u , d , and s can be quite well described by a semirelativistic potential model including instanton induced forces. The spectra of baryons composed of the same quarks is studied using the same model. The results and the limitations of this approach are described. Some possible improvements are suggested.

I. INTRODUCTION

The QCD based semirelativistic potential model is a successful approach to describe both meson and baryon spectra. In most of these works, it is assumed that the quark interaction is dominated by a linear confinement potential, and that a residual interaction stems from the one-gluon exchange mechanism. In particular, the spin-spin interaction implied by this process is responsible of the non-degeneracy of π and ρ mesons. The results obtained with such models are generally in good agreement with experimental data, but the mesons η and η' cannot be described without adding an appropriate flavor mixing interaction.

Another QCD based candidate exists for the residual interaction: The effective forces computed by 't Hooft from instanton effects [1]. It is a pairing force which presents the peculiarities to act only on quark-antiquark states with zero spin and zero angular momentum in the nonrelativistic limit; it also generates constituent masses for the light quarks. A flavor mixing appears naturally with this interaction which has already been used in various models to study light mesons: Nonrelativistic potential model [2], instantaneous Bethe-Salpeter formalism [3, 4], flux-tube model [5], semirelativistic potential approach [6, 7]. In all cases, quite good results are obtained.

A first attempt to test this instanton induced forces for baryons is performed in Ref. [2]. With the same set of parameters, a description of all light mesons and baryons is obtained (only the constant potential is changed from mesons to baryons). The ground states of spectra are quite well reproduced but some meson and baryon excitations are obtained too high: Vibrational excitations of K and K^* mesons, η' , ϕ , vibrational excitations of N , Λ , etc. Moreover, this model presents two serious drawbacks. First, it is a nonrelativistic model. As the velocity of a light quark inside a hadron is not small compared with the speed of light, the interpretation of the parameters of the model is questionable. Second, the constituent masses and the coupling constants of the instanton induced forces have been considered as free parameters fitted to reproduce at best meson spectra. Actually, these quantities can be calculated from instanton theory.

More recently, an instantaneous Bethe-Salpeter three-body formalism has been applied to the study of baryons with instanton induced forces [8, 9, 10]. In these works, the three-body generalization of the Hamiltonian developed in Ref. [4] is used, but with parameters fitted for the baryons, that is to say different from those found for the mesons. The constituent

masses and the coupling constants of the instanton induced forces are also considered as free parameters. The results of these works are compared with our results in the following.

In a previous work [11], we have developed a semirelativistic model for mesons including the instanton induced forces, but with parameters calculated, as far as possible, with the underlying theory. Very good results have been obtained with the condition that the quarks are considered as effective degrees of freedom with a finite size. In particular, all ground states of vector and pseudoscalar mesons are well reproduced, generally better than in Ref. [2]. The main flaw stems from the usual problem of the vibrational excitations of K and K^* mesons. In this paper, we use this model to describe baryons composed of u , d , and s quarks, in order to test the relevance of the instanton induced forces in the framework of a semirelativistic potential model.

Note that for some authors [12] the pion should be treated as a pseudo Goldstone boson and not as a quark-antiquark state. Nevertheless, in Refs. [2, 3, 4, 5, 6, 7, 11] devoted to the study of mesons with instanton induced forces, the pion can always be obtained with a correct mass. We think that this is not by chance since, with the resulting pion wave function, it is possible to compute the correct pion charge form factor [13], reasonable values for the electromagnetic splittings [11], and (more convincing) correct hadronic decay widths in which pions are produced [14]. We believe that the instanton induced interaction can simulate processes giving to the pion its very low mass, and that it is relevant to fix the parameters of the Hamiltonian in order to obtain all pseudoscalar mesons.

The main characteristics of the model are recalled in Sec. II. The numerical technique used to compute the baryon masses and the fitting procedures are briefly described in Sec. III, where various baryon spectra obtained are also discussed. Some concluding remarks are given in Sec. IV.

II. MODEL

The model we use is the natural generalization to baryon of the Hamiltonian built for mesons in Ref. [11]. It is worth noting that this Hamiltonian is defined at the lowest order, that is to say that none relativistic correction is included in the potential. Details can be found in Ref. [11].

The three-quark Hamiltonian is written

$$H = \sum_{i=1}^3 \sqrt{\vec{p}_i^2 + m_i^2} + \sum_{i<j=1}^3 V_{ij}, \quad (1)$$

with \vec{p}_i the momentum of quark i ($\sum_{i=1}^3 \vec{p}_i = \vec{0}$), m_i its constituent mass, and V_{ij} the interaction between quarks i and j . The interaction contains the Cornell potential and the instanton interaction. The Cornell potential, which depends only on distance r between two quarks, is given by

$$V_C(r) = \frac{1}{2} \left[-\frac{\kappa}{r} + a r + C \right], \quad (2)$$

where the $1/2$ factor takes into account the color contribution. The confining part of this potential represents a good approximation of the Y-shape string configuration.

The instanton induced interaction provides a suitable formalism to reproduce well the spectrum of the pseudoscalar mesons and to explain the masses of the η - and η' -mesons. In the nonrelativistic limit, this interaction between two quarks in a baryon is written [2, 3]

$$V_I(r) = -4 \left(g P^{[nn]} + g' P^{[ns]} \right) P^{S=0} \delta(\vec{r}), \quad (3)$$

where g, g' are two dimensioned constants, $P^{S=0}$ is the projector on spin 0, and $P^{[qq']}$ is the projector on antisymmetrical flavor state qq' (n for u or d is a non-strange quark, and s is the strange quark). The operator $P^{[nn]}$ is simply a projector on isosinglet states, but the operator $P^{[ns]}$ is not so easy to implement. Indeed, the instanton interaction is obtained under the hypothesis of a perfect SU(3) flavor symmetry [2, 3]. So, the baryon wave function is assumed to have a definite spin-flavor symmetry, as in the simple model of Ref. [15] used to calculate baryon mass splitting. Within a more realistic model, the strange quark is much heavier than a n -quark, and the wave function cannot have a particular flavor symmetry other than an isospin symmetry for the n -quarks. Consequently, the flavor matrix elements $\langle ns | P^{[ns]} | ns \rangle$ have values in our model which are half the values in Ref. [15]. To be compatible with this reference, we have placed a supplementary factor 2 in front of the operator $P^{[ns]}$ in the computation code. The procedure to handle the same problem in Ref. [2] is not described.

The instanton induced forces also give a contribution Δm_q to the current quark mass m_q^0 . As this interaction is not necessarily the only source for the constituent mass, a phenomenological term δ_q is also added to the current mass [11]. Finally, the constituent masses in our

models are given by

$$m_n = m_n^0 + \Delta m_n + \delta_n, \quad (4)$$

$$m_s = m_s^0 + \Delta m_s + \delta_s. \quad (5)$$

In the instanton theory, the quantities g , g' , Δm_n , Δm_s are given by integrals over the instanton size ρ up to a cutoff value ρ_c (see for instance Ref. [3, formulas (5) to (9)]). These integrals can be rewritten in a more interesting form for numerical calculations by defining a dimensionless instanton size $x = \rho\Lambda$ where Λ is the QCD scale parameter [11]

$$g = \frac{\delta \pi^2}{2} \frac{1}{\Lambda^3} \left[m_s^0 \alpha_{11}(x_c) - \frac{c_s}{\Lambda^2} \alpha_{13}(x_c) \right], \quad (6)$$

$$g' = \frac{\delta \pi^2}{2} \frac{1}{\Lambda^3} \left[m_n^0 \alpha_{11}(x_c) - \frac{c_n}{\Lambda^2} \alpha_{13}(x_c) \right], \quad (7)$$

$$\Delta m_n = \delta \frac{1}{\Lambda} \left[m_n^0 m_s^0 \alpha_9(x_c) - \frac{(c_n m_s^0 + c_s m_n^0)}{\Lambda^2} \alpha_{11}(x_c) + \frac{c_n c_s}{\Lambda^4} \alpha_{13}(x_c) \right], \quad (8)$$

$$\Delta m_s = \delta \frac{1}{\Lambda} \left[(m_n^0)^2 \alpha_9(x_c) - 2 \frac{c_n m_n^0}{\Lambda^2} \alpha_{11}(x_c) + \frac{(c_n)^2}{\Lambda^4} \alpha_{13}(x_c) \right], \quad (9)$$

with

$$\alpha_n(x_c) = \int_0^{x_c} dx \left[9 \ln \left(\frac{1}{x} \right) + \frac{32}{9} \ln \left(\ln \left(\frac{1}{x} \right) \right) \right]^6 x^n \left(\ln \left(\frac{1}{x} \right) \right)^{-32/9}. \quad (10)$$

In these equations, $\delta = 3.63 \cdot 10^{-3} \times 4\pi^2/3$ and $c_i = (2/3)\pi^2 \langle \bar{q}_i q_i \rangle$ where $\langle \bar{q}_i q_i \rangle$ is the quark condensate for the flavor i . Except the quantity $x_c = \rho_c \Lambda$, all parameters involved in Eqs. (6)–(9) have expected values from theoretical and/or experimental considerations. The integration in Eq. (10) must be carried out until the ratio of the \ln -term on the $\ln \ln$ -term into the integral stays small [2, 11]. This ratio increases with x from zero at $x = x_1 = 1/e$ to very large values. At $x = x_2 \approx 0.683105$, the value of this ratio is 1. This last value corresponds to the minimum of the instanton density (see Ref. [11, Fig. 1]). Thus we define the parameter ϵ by

$$\rho_c = x_c / \Lambda \quad \text{with} \quad x_c = x_1 + \epsilon(x_2 - x_1) \quad \text{and} \quad \epsilon \in [0, 1]. \quad (11)$$

In this work ϵ is a pure phenomenological parameter whose value must be comprised between 0 and 1. With this procedure, the value of the cutoff instanton size in our model is comprised between 0.3 and 0.5 fm, which is a reasonable range for this parameter [2, 10].

The quark masses used in our model are the constituent masses and not the current ones. It is then natural to suppose that a quark is not a pure point-like particle, but an effective

degree of freedom which is dressed by the gluon and quark-antiquark pair clouds. The form that we retain for the probability density of a quark is a Gaussian function

$$\rho_i(\vec{r}) = \frac{1}{(\gamma_i \sqrt{\pi})^{3/2}} \exp(-r^2/\gamma_i^2). \quad (12)$$

It is generally assumed that the quark size γ_i depends on the flavor. So, we consider two size parameters γ_n and γ_s for n and s quarks respectively. It is assumed that the dressed expression $\tilde{O}_{ij}(\vec{r})$ of a bare operator $O_{ij}(\vec{r})$, which depends only on the relative distance $\vec{r} = \vec{r}_i - \vec{r}_j$ between the quarks q_i and q_j , is given by

$$\tilde{O}_{ij}(\vec{r}) = \int d\vec{r}' O_{ij}(\vec{r}') \rho_{ij}(\vec{r} - \vec{r}'), \quad (13)$$

where ρ_{ij} is also a Gaussian function of type (12) with the size parameter γ_{ij} given by

$$\gamma_{ij} = \sqrt{\gamma_i^2 + \gamma_j^2}. \quad (14)$$

This formula is chosen because the convolution of two Gaussian functions, with size parameters γ_i and γ_j respectively, is also a Gaussian function with a size parameter given by Eq. (14).

After convolution with the quark density, the Cornell dressed potential has the following form

$$\tilde{V}_C(r) = -\kappa \frac{\text{erf}(r/\gamma_{ij})}{r} + a r \left[\frac{\gamma_{ij} \exp(-r^2/\gamma_{ij}^2)}{\sqrt{\pi} r} + \left(1 + \frac{\gamma_{ij}^2}{2r^2}\right) \text{erf}(r/\gamma_{ij}) \right] + C, \quad (15)$$

while the Dirac-distribution in $V_I(r)$ is transformed into a Gaussian function

$$\tilde{\delta}(\vec{r}) = \frac{1}{(\gamma_{ij} \sqrt{\pi})^3} \exp(-r^2/\gamma_{ij}^2). \quad (16)$$

Despite this convolution, we consider, for simplicity, that the instanton induced forces act always only on $L = 0$ states. Note that the strange size quark can be vanishing provided the non-strange quark size is non-zero. Indeed, $\gamma_s = 0$ with $\gamma_n \neq 0$ yields $\gamma_{nn} \neq 0$, $\gamma_{ns} \neq 0$; only $\gamma_{ss} = 0$. This last value could pose a problem only in expression (16). But this situation never happens since the instanton interaction V_I is vanishing for a ss pair (see Eq. (3)). For mesons, the situation is a little bit more complicated and it is discussed in Ref. [11].

III. NUMERICAL RESULTS

A. Numerical technique

The eigenvalue equation is solved by developing the wave functions in trial states built with harmonic oscillator states $|nlm\rangle$. In such a basis, the two-body matrix elements of the potential are expressed in terms of the following quantities

$$\langle n'lm|V(r)|nlm\rangle = \sum_{p=l}^{l+n+n'} B(n', l, n, l, p) I_p, \quad (17)$$

with

$$I_p = \frac{2}{\Gamma(p+3/2)} \int_0^\infty dx \, x^{2p+2} \exp(-x^2) V(bx). \quad (18)$$

The quantities I_p are the Talmi's integrals, which depend on a nonlinear parameter b , the oscillator length. The coefficients $B(n', l, n, l, p)$ are geometric factors [16] which can be calculated once for all. To accelerate the convergence, we use two oscillator lengths b, b' in our basis. These two quantities are the scale parameters of the two internal radial distances which can be defined in a baryon. This method, which has originally been developed in Ref. [17] for nonrelativistic kinematics, works very well for relativistic kinematics, as it is shown in Ref. [18]. The details of the technique used to calculate the matrix elements of the relativistic kinetic energy operator can be also found in Ref. [18].

B. Fitting procedure

The purpose of this work is to extend to the baryons the results obtained for the meson spectra in Ref. [11], and thus try to obtain a satisfactory description of baryon spectra with a quite simple model. Our approach is indeed very simple since we use only a spinless Salpeter equation supplemented by a pure central potential and the nonrelativistic limit of an instanton induced interaction, that is to say that the potential is completely defined at the zero order of quark speed. This is sufficient to describe the bulk properties of mesons and baryons. Note that the instanton induced interaction is essential to describe pseudoscalar mesons and the baryon ground state properties, such as the N - Δ splitting.

We need thirteen parameters to obtain a satisfactory spectrum for the mesons, and to be consistent, we keep the same set of parameters for the baryons. This number could appear

large, but some are strongly constrained by theoretical or experimental considerations, while other are unavoidable (see discussion in Ref. [11]). The instanton interaction is defined by six parameters: The current quark masses, the quark condensates for the flavors n and s , the QCD scale parameter Λ , and the maximum size ρ_c (x_c/Λ) of the instanton. Four other parameters are introduced: The effective sizes of the quarks n and s , and two terms δ_n and δ_s , which contribute to the constituent quarks masses. It is worth mentioning that, among the above parameters, m_n^0 , m_s^0 , Λ , c_n , c_s , δ_n , δ_s , and ϵ are intermediate quantities used to compute the four parameters m_n , m_s , g , and g' which enter directly into the Hamiltonian. Three unavoidable parameters are also used for the central part of the potential: The slope of the confinement a —for which reliable estimations exist—, the strength κ for the Coulomb-like part, and the constant C which renormalizes the masses. Consequently, we can say that only six quantities are really free parameters (see Table II).

To find the value of the parameters, we have minimized a χ^2 function based on the masses of 11 well-known baryons (see Table I)

$$\chi^2 = \sum_i \left[\frac{M_i^{\text{th}} - M_i^{\text{exp}}}{\Delta M_i^{\text{exp}}} \right]^2, \quad (19)$$

where the quantity ΔM_i^{exp} is the error on the experimental masses (it is fixed at the minimum value of 10 MeV, see Ref. [11]). To perform the minimization, we use the most recent version of the MINUIT code from the CERN library [19].

C. Baryon spectra

We first compute baryon spectra with the parameters found in our previous paper for meson spectra [11], but the results obtained are not very good. For instance, the roper resonance is found 576 MeV above the nucleon—which is not so bad—, but the N – Δ mass difference computed is 212 MeV, which is much too small. This mass difference is generally considered as a minimum requirement to be reproduced for a baryon model.

In a second step, we have searched for a set of parameters to describe both baryon and meson spectra. All sets found are very similar and present more or less the same qualities and the same flaws (best results are obtained with the supplementary factor 2 in front of the operator $P^{[ns]}$). For instance, in one of the best sets of parameters found, a good N – Δ mass difference is obtained (280 MeV), but the Roper resonance mass is then calculated

around 150 MeV above its experimental value. Moreover, if the meson spectra obtained do not differ significantly from the ones found in our previous paper [11], two states are then very badly described with respect to the others: The η' -meson is found 36 MeV too high and the 3D_J states are computed 43 MeV too low. Despite a great number of minimizations, we never succeeded to find an “acceptable” set of parameters to describe satisfactorily both meson and the baryon sectors.

In order to test the relevance of our model for baryons, we have then searched for parameters to describe baryons only. One of the best set of baryon spectra that we have found is given in Fig. 1 to Fig. 5. The spectra present some characteristics which can be found in several other works, in particular Ref. [20]; only few states are not so well reproduced in our work. For example, the mass of the Roper resonance is around 60 MeV too high. The nucleon states with negative parity have masses which are slightly too small. The Roper of the Δ has a too high mass. The $\Lambda(1405)$ cannot be described, as this is often the case. Even if our spectra are clearly less good than the spectra found in Ref. [20], they presents many similar qualitative characteristics. But in general, the agreement between calculated masses and experimental data is less good in our model. It is worth noting that all the baryon ground states can be well reproduced. With these parameters fitted to the baryons, the mesons masses are very poorly obtained: For instance, if the computed pion mass is good (138 MeV), the mass of the ρ -meson is found 260 MeV above its experimental value. The impossibility to obtain good meson and baryon spectra with the same parameters is also a characteristic of the model of Ref. [20].

It is also interesting to compare our spectra with those obtained in Refs. [8, 9, 10]. The model developed in these works and our model are similar in the sense that the instanton induced interaction is the only spin-isospin-dependent part of the Hamiltonian, but the model of Refs. [8, 9, 10] differs from ours by two main points: *i)* the use of a spinless Salpeter equation in our model instead of an instantaneous Bethe-Salpeter equation, *ii)* the presence of a Coulomb-like interaction in our model. Below 2 GeV, spectra of both models are very similar; they share more or less the same qualities and the same flaws: the ground states are well reproduced, but the Roper resonance and the first $J^P = 1/2^-$ state are inverted. Again the $\Lambda(1405)$ cannot be described. We can just note a slight improvement for other Λ -baryons. The baryon Regge trajectories are nevertheless better described in the model of Refs. [8, 9, 10]; this is an indication of the better relevance of an instantaneous

Bethe-Salpeter equation over a simpler spinless Salpeter equation. Note that our model is characterized by smaller values of parameters g and g' . This is due to the fact that the instanton interaction can be weaker in our model since we include in our Hamiltonian an attractive Coulomb-like interaction.

As one can see from Table II, the values of the seven first parameters are rather satisfactory (ϵ is expected to be near zero, see Ref. [11]), while one can see that the Coulomb-like parameter κ is rather strong and that the size of the s -quark is almost zero. A so small value for the strange quark size could appear troublesome but this does not cause any numerical difficulties, as mentioned above. Good meson and baryon spectra can be computed with γ_s around $0.5\gamma_n$ and reasonable values for the parameters m_s^0 and δ_s . But the better baryon spectra are obtained with small values of γ_s . As it is expected we found $g' < g$ [9]. Note that when the parameters are fitted only to baryons, the factor 2 in front of the operator $P^{[ns]}$ can be simulated by a redefinition of the parameter g' . But in this case, we have $g' > g$. It is also clear from this Table that some of the best parameters for baryons are different from the corresponding best parameters for mesons, in particular quantities related to the quark masses. Moreover, the value of the parameter κ is higher for baryon, while the value of the confinement slope a is lower. With so much differences in these two sets of parameters, it is not surprising that mesons and baryons cannot be well reproduced together. Some physics is clearly missing. We discuss this point in the next section.

IV. CONCLUDING REMARKS

Several works have been devoted to the study of the instanton interaction in the framework of semirelativistic or relativistic models for mesons [3, 4, 5, 6, 7, 8, 9, 10]. A few of these models have been applied to baryons. Our purpose was here to compute baryon spectra with the semirelativistic instanton based model for mesons we have developed in Ref. [11] and with the same underlying fundamental ingredients. When this model is directly applied to baryons, the spectra obtained are not good. It is necessary to change all the parameters to compute a more relevant spectra. The natural link between meson and baryon is then broken, and the baryon spectra obtained are not very different from those yielded by models *i)* with similar complexity but based on one-gluon exchange process (see for instance Ref. [21]), *ii)* relying on covariant equation with only an instanton induced

interaction supplementing the confinement [8, 9, 10].

Our semirelativistic model Hamiltonian contains a potential part written in the lowest order, that is to say that none relativistic correction is included. In particular, the spin-spin term—responsible of the low pion mass in most potential models (see for instance Ref. [22])—is not present here. The instanton induced forces are assumed to take into account all spin effects. This is probably a too crude approximation. Both interactions, instanton induced one and spin-spin term, have very similar contributions for non-mixed flavor mesons. Nevertheless, to include the two interactions in a Hamiltonian means a complete new fitting of parameters, in particular the parameter κ which measures the strength of the Coulomb-like potential and of the spin-spin interaction. This could modify appreciably the spectra of baryons. It is not sure that the inclusion of relativistic corrections in the model could cure all its defects. In particular, the relative position of positive and negative parity excitations of the nucleon is a problem for all models based on the one-gluon exchange dominance. This puzzle is solved with the meson exchange potential proposed more recently [20]. Within this model, the quarks interact by exchanging pseudoscalar mesons, completely ruling out the one-gluon exchange process. Despite some serious critics [23], one is forced to ascertain that spectra of light baryons are remarkably improved. It is thus possible that the meson exchange process could be one of the key to explain baryon spectra and could supplement instanton induced interaction. Such a study is in progress. Let us note that the Hamiltonian described in Ref. [20] cannot reproduce meson and baryon spectra with the same set of parameters. It thus suffers the same drawback than our model.

This work clearly shows that the instanton induced forces cannot explain alone both meson and baryon spectra. Contributions coming from one-gluon and meson exchange processes are probably necessary. Nevertheless, as the 't Hooft interaction solves naturally the π - K - η - η' problem without any additional assumptions, it must certainly be an unavoidable ingredient of potential models.

Acknowledgments

C. Semay would thank the F.N.R.S. for financial support. F. Brau would thank the I.I.S.N. for financial support.

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TABLE I: Quantum numbers and masses (the minimal uncertainty is fixed at 10 MeV, see Ref. [11]) of the baryons used in the minimization procedure to find the parameters listed in Table II.

Baryon	I	J^P	Masses (GeV)
N	$\frac{1}{2}$	$\frac{1}{2}^+$	0.939 ± 0.010
$N(1440)$	$\frac{1}{2}$	$\frac{1}{2}^+$	1.450 ± 0.020
Δ	$\frac{3}{2}$	$\frac{3}{2}^+$	1.232 ± 0.010
$N(1535)$	$\frac{1}{2}$	$\frac{1}{2}^-$	1.537 ± 0.018
Λ	0	$\frac{1}{2}^+$	1.116 ± 0.010
Σ	1	$\frac{1}{2}^+$	1.193 ± 0.010
Σ^*	1	$\frac{3}{2}^+$	1.385 ± 0.010
Ξ	$\frac{1}{2}$	$\frac{1}{2}^+$	1.315 ± 0.010
Ξ^*	$\frac{1}{2}$	$\frac{3}{2}^+$	1.530 ± 0.010
Ω	0	$\frac{3}{2}^+$	1.672 ± 0.010
$\Delta(1600)$	$\frac{3}{2}$	$\frac{3}{2}^+$	1.625 ± 0.075

TABLE II: List of parameters of the Model. The column “Baryon” contains the values for the baryon spectra presented in Fig. 1 to Fig. 5. The column “Meson” contains the values for the meson model I of Ref. [11]. When available, the expected value of a parameter is also given in the column “Exp.”. The values of the quantities m_n , m_s , g , and g' computed with these parameters are also indicated.

Parameters	Unit	Baryon	Meson	Exp.
m_n^0	GeV	0.001	0.015	0.001–0.009 [24]
m_s^0	GeV	0.103	0.215	0.075–0.170 [24]
Λ	GeV	0.238	0.245	$0.208^{+0.025}_{-0.023}$ [24]
$\langle \bar{n}n \rangle$	GeV ³	$-(0.247)^3$	$-(0.243)^3$	$(-0.225 \pm 0.025)^3$ [25]
$\langle \bar{s}s \rangle / \langle \bar{n}n \rangle$		0.631	0.706	0.8 ± 0.1 [25]
ϵ		0.061	0.031	0–1 [11]
a	GeV ²	0.168	0.212	0.20 ± 0.03 [26]
κ		0.798	0.440	
C	GeV	−0.967	−0.666	
γ_n	GeV ^{−1}	0.681	0.736	
γ_s	GeV ^{−1}	0.005	0.515	
δ_n	GeV	0.327	0.120	
δ_s	GeV	0.490	0.173	
m_n	GeV	0.378	0.192	
m_s	GeV	0.638	0.420	
g	GeV ^{−2}	2.498	2.743	
g'	GeV ^{−2}	2.234	1.571	

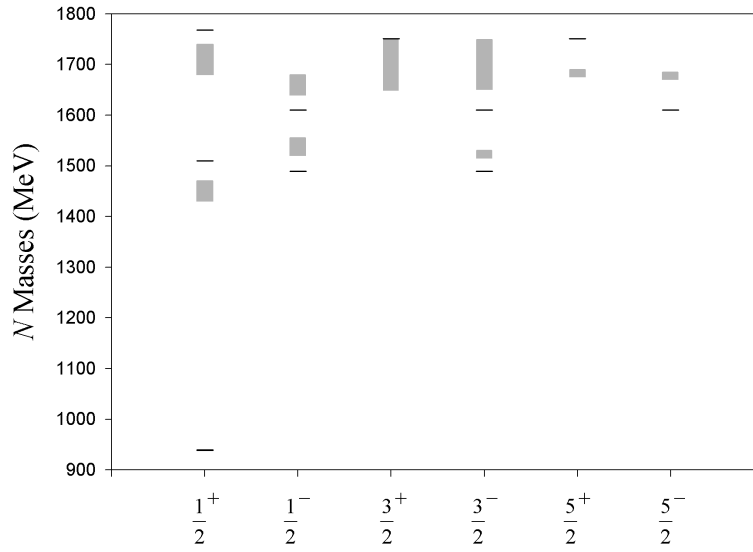


FIG. 1: Energy levels of the nucleon states (status ★★★ and ★★) as a function of total angular momentum and parity J^P . The shaded boxes represent the experimental values with the uncertainties.

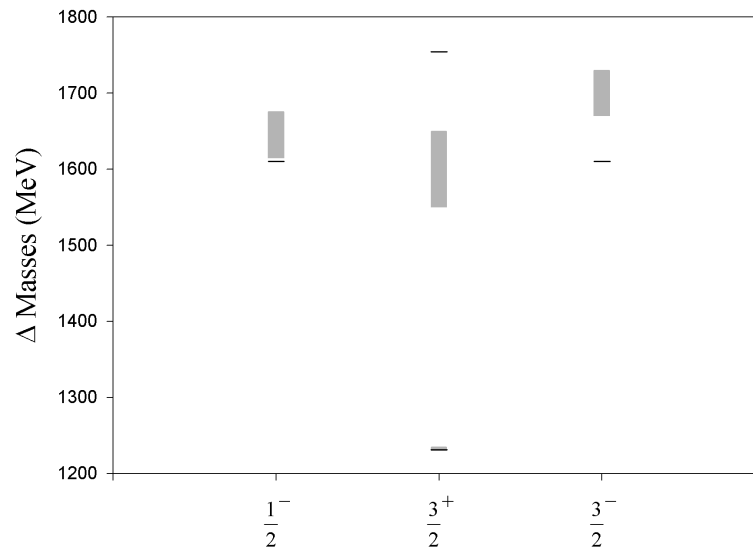


FIG. 2: Same as Fig. 1 but for the Δ states.

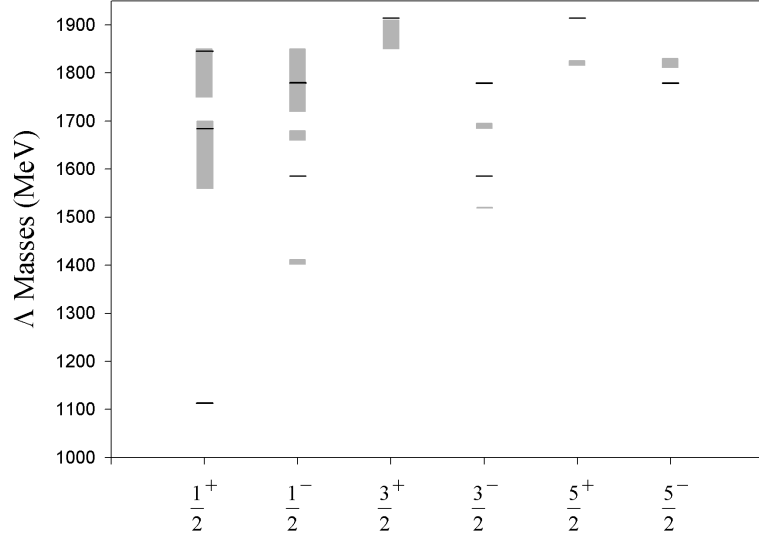


FIG. 3: Same as Fig. 1 but for the Λ states.

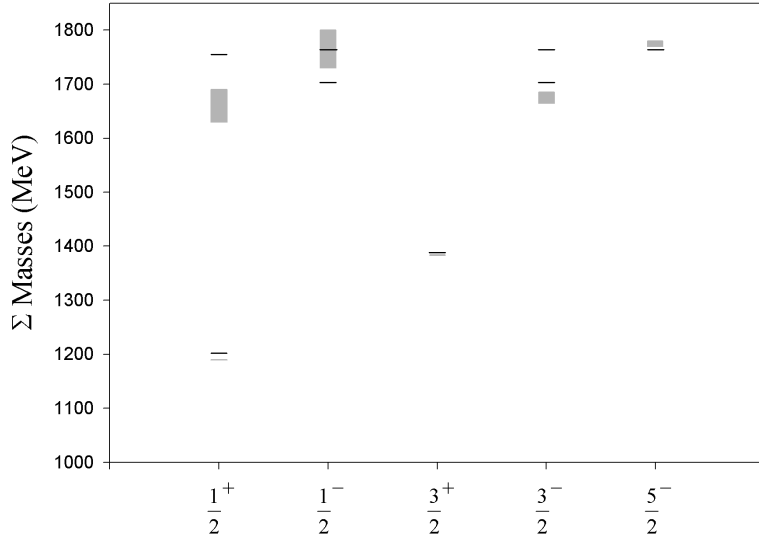


FIG. 4: Same as Fig. 1 but for the Σ states.

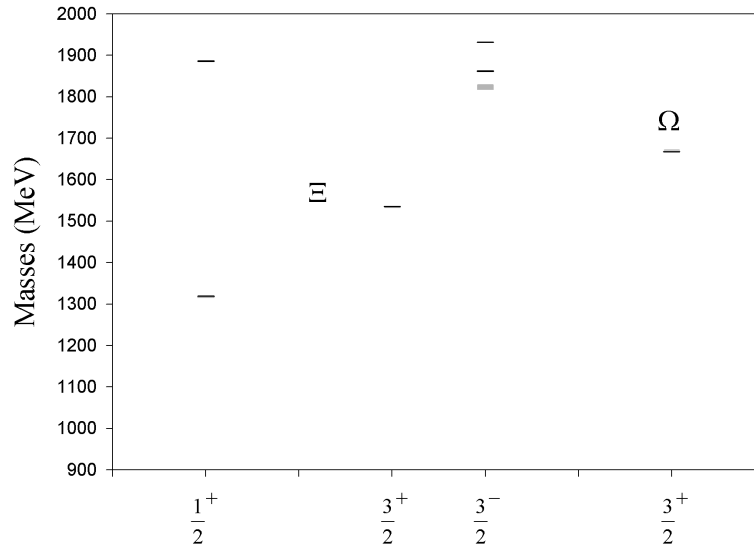


FIG. 5: Same as Fig. 1 but for the Ξ and Ω states.